REMOTELY SENSING RUBIDIUM WITH ABSORPTION SPECTROSCOPY

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Aim: To remotely sense the number density of rubidium using absorption spectroscopy targeting the D2 transition line

THEORY

Beer-Lambert Law

The Beer-Lambert Law describes the light intensity reduction by the absorption cross section (σ), number density (N), and path length (l). I_0 and I_f represents the light intensity that enters and exits the rubidium cell respectively.

$$\ln\left(\frac{I_0}{I_f}\right) = \sigma N l \ [3] \ \text{----- (Eqn. 1)}$$

Doppler Linewidth

Atoms observe a frequency shift (Δf) from the laser's frequency (f) proportional to its velocity (v) due to the doppler effect. Doppler broadening as a function of detuning is Gaussian [2].

$$G(f_{detuning}) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(f_{detuning})^2}{2\sigma^2}} \ \text{----- (Eqn. 2)}$$

Natural Linewidth

The natural linewidth [1,4] quantifies the small range of frequencies around resonance than atoms can absorb for excitation at zero temperature.

$$L(f_{detuning}) = \frac{1}{1 + 4\left(\frac{f_{detuning}}{6.0659 MHz}\right)^2} \ \text{----- (Eqn. 3)}$$

Absorption Cross Section as a Function of Detuning

$$\sigma(f_{detuning}) = \sigma_o V(f_{detuning}) \ \text{----- (Eqn. 4)}$$

The absorption cross section (σ) at resonance [1,4] varies with the detuning of light by the Voigt profile ($V(f_{detuning})$), accounting for doppler and natural linewidths. The Voigt profile is expressed below, with f' the variable of integration. g is defined to be central frequency of the hyperfine transition (where $f_{detuning} = 0$)

Convolving equations 2 and 3, $V(g) = \int L(f') * G(g - f') \ df' \ \text{----- (Eqn. 5)}$

Saturation Effects

As laser power decreases, the proportion of ground state atoms increases. At sufficiently low powers, the measured number density of ground state atoms can be approximated to be the total number density of rubidium.

Total Number Density (Objective)

Combining equations 1, 4 and 5, the total number density of rubidium can be given as $N = \frac{\ln\left(\frac{I_0(g')}{I_f(g')}\right)}{\sigma l V(g)}$, where g' is the central frequency of the transition where detuning = 0
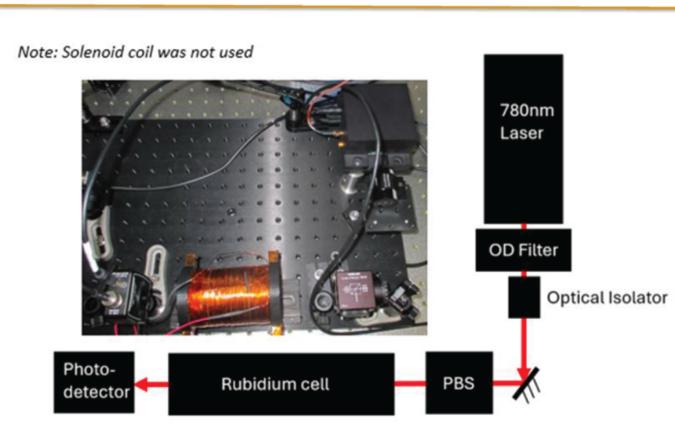
Empirical Gas Model

The Clausius-Clapeyron equation [5] predicts vapour pressure over a physical state. The number density ($[A]$) is then given according to the ideal gas law as a function of temperature (T).

$$[A] = [A]_o \left(\frac{T_o}{T}\right) e^{\left(b\left[1 - \frac{T_o}{T}\right]\right)}$$

At 297.75 Kelvin, the number density was estimated as 10^{10} atoms/cm³.

METHODOLOGY

Note: Solenoid coil was not used

780nm Laser — OD Filter — Optical Isolator — PBS — Rubidium cell — Photo-detector

A 1mm radius laser beam targeted the D2 line. An oscilloscope was used to modulate the frequency of the laser by varying the input current. The rubidium cell was enclosed by a solenoid coil, though not used in this study. A photodetector measured the output light intensity.

To Determine I_0

Cell was removed from the setup, and optical density (OD) filters were placed between the laser and optical isolator. → An amplifier was used to add gain to the weak absorption spectrums. → The data from the photodetector was recorded while a light power meter was also used to measure the averaged power of the sweep.

Measurements were repeated as combinations of OD filters with increasing magnitudes were used (OD 2.0 to 5.9)
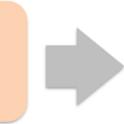
To Determine I_f

Cell was placed between the beam splitter and photodetector. OD filters were placed between the laser and optical isolator. → Same gain settings were applied to th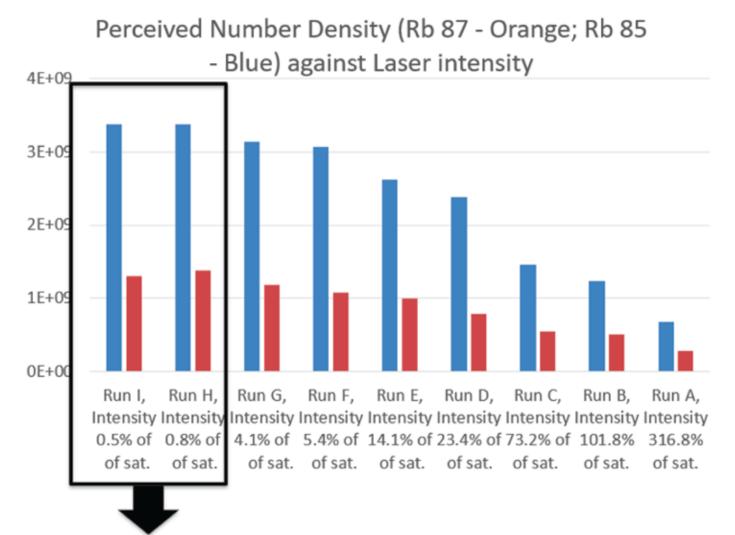e amplifier according to laser power. → The data from the photodetector was recorded. → The temperature of the rubidium cell was determined by placing a temperature probe against the rubidium cell.
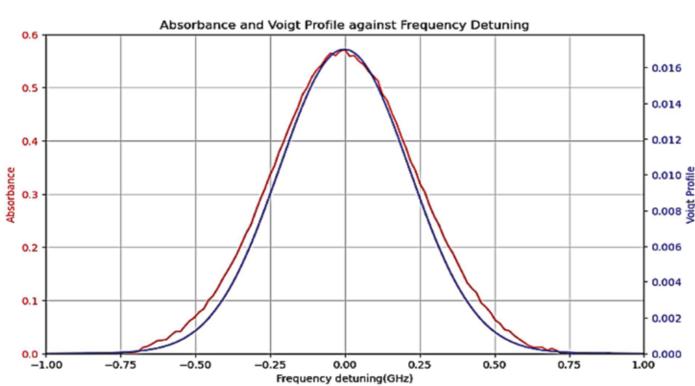
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RESULTS/DISCUSSION

Number Density (Spectroscopy)

As incident light intensity increased, the number density of rubidium was perceived to decrease. Hence, only runs with laser intensities attenuated by OD filters to be under 1% of the saturation threshold (2.5033mW/cm²) were analysed.

Perceived Number Density (Rb 87 - Orange; Rb 85 - Blue) against Laser intensity

| | Run I, Intensity 0.5% of sat. | Run H, Intensity 0.8% of sat. | Run G, Intensity 4.1% of sat. | Run F, Intensity 5.4% of sat. | Run E, Intensity 14.1% of sat. | Run D, Intensity 23.4% of sat. | Run C, Intensity 73.2% of sat. | Run B, Intensity 101.8% of sat. | Run A, Intensity 316.8% of sat. |

Absorbance as a Voigt Profile

Absorbance and Voigt Profile against Frequency Detuning

Absorbance ($\ln\left(\frac{I_0}{I_f}\right)$) (red) and theoretical Voigt profile (black) are plotted against detuning for a hyperfine peak. The experimental data (Absorbance) follows a similar trend compared to the theoretical model (Voigt Profile).

Comparing Spectroscopy with the Empirical Model

Despite being within an order of magnitude, the ideal gas law's estimate was significantly greater than absorption spectroscopy's.

Potential explanations

The empirical model (1995 CRC) used, just like other models, have a large degree of error [5]. For example, at 20 degrees Celsius, the Killian and Nesmeyanov models deviate by 7% and 25% from the CRC model.
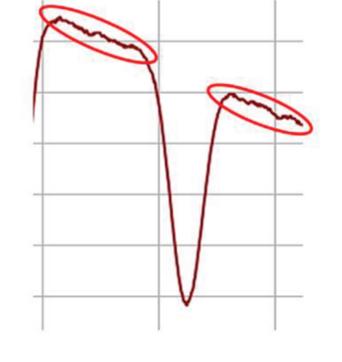
Potentials and Limitations of this Technique

✓ Allows for measurements of each isotope by measuring the absorbance of each hyperfine peak.

✓ Likely to work in the presence of other atoms as long as its resonant peaks (if any) does not interfere with rubidium.

✗ Only can be used to quantify low concentrations, where the linear relationship between absorbance and number density works.

✗ Cell must have minimal and uniform reflective and absorption properties across frequencies probed.

Potential Sources of Error

Noise due to light interference and laser imperfections is a source of error. However, the effect of this is minimal, with this error affecting the averaged total number density by a mere $\pm 1.9 \times 10^7$ atoms/cm³.

A small variance in temperature results in large change in number density. The fact that temperature readings were not measured during the spectroscopy, but after the experiments were over gives rise to large margin of error.

Run	Laser Intensity (% of saturation)	Number Density of Rb 85 (cm⁻³)	Number Density of Rb 87 (cm⁻³)	Overall Number Density (cm⁻³)
H	0.8	$3.37 \times 10^9 \pm 2.7 \times 10^7$	$1.39 \times 10^9 \pm 2.7 \times 10^7$	$4.76 \times 10^9 \pm 3.8 \times 10^7$
I	0.5	$3.38 \times 10^9 \pm 2.7 \times 10^7$	$1.30 \times 10^9 \pm 2.7 \times 10^7$	$4.68 \times 10^9 \pm 3.8 \times 10^7$

Averaging these results, the overall number density is $4.72 \times 10^9 \pm 1.9 \times 10^7$ atoms/cm³

References/Acknowledgements

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[3] Beer-lambert law with Napierian Absorbance: Wolfram Formula Repository. Beer–Lambert Law with Napierian Absorbance | Wolfram Formula Repository. (n.d.-b). https://resources.wolframcloud.com/FormulaRepository/resources/BeerLambert-Law-with-Napierian-Absorbance

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[5] Singh, J., Dolph, P. A. M., & Tobias, W. A. (2008, April 23). Alkali Metal Vapor Pressures & Number Densities for Hybrid Spin Exchange Optical Pumping. https://people.frib.msu.edu/~singhj/docs/vp195.pdf

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